Field-dependent nonlinear surface resistance and its optimization by surface nano-structuring of SRF cavities

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Why are Q-E curves so different?
Unfortunately, the Mattis-Bardeen’s theory for the weak-field $R_s$ tells you nothing about the shape of $Q$-$E$ curve.

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- It is valid only at the weak-field limit: the left end of Q-E curve.
- Effects of material features other than mfp are not taken into account: thus may not be valid even at the weak-field limit.

Need to develop a theory of field dependent $R_s$ including realistic material features.
Part 1: Review

Effects of density of states (DOS) broadening

- A. Gurevich,

- A. Gurevich and T. Kubo,
The surface resistance is given by

\[ R_s = \frac{1}{2} \mu_0^2 \omega^2 \lambda^3 \sigma_1 \]

Here \( \sigma_1 \) is roughly (when \( T \ll T_c \) and \( \omega \ll T \))

\[ \sigma_1 \sim \sigma_n \int_{\Delta}^{\infty} N(\epsilon)N(\epsilon + \hbar \omega)e^{-\frac{\Delta}{kT}}d\epsilon \]

**Weak-field limit**

Mattis Bardeen’s formula

\[ \sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar \omega} e^{-\frac{\Delta}{kT}} \]

comes from this DOS

However, it is well known that the DOS is affected by the pair-breaking current.

DOS under a dc current

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\[ \sigma_1 \approx \sigma_n \int_{\Delta}^{\infty} N(\epsilon) N(\epsilon + \hbar \omega) e^{-\frac{\Delta}{kT}} d\epsilon \]

**Weak-field limit**

**Idealized BCS DOS**

\[ \frac{N(\epsilon)}{N_0} = \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2}} \]

**Mattis Bardeen’s formula**

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**However, it is well known**

**that the DOS is affected by the pair-breaking current.**
Review (7)

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**Weak-field limit**

Idealized BCS DOS

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\[ \sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar \omega} e^{-\frac{\Delta}{kT}} \]

This logarithmic factor in MB’s formula comes from the sharp peak of the idealized BCS DOS

Under a strong rf current

DOS peaks oscillate (animation)

Mattis Bardeen’s formula

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$$\sigma_1 \sim \sigma_n \int_{\Delta}^{\infty} N(\epsilon)N(\epsilon + \hbar \omega)e^{-\Delta/\kappa T} d\epsilon$$

**Weak-field limit**

**Idealized BCS DOS**

$$\frac{N(\epsilon)}{N_0} = \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2}}$$

**Under a strong rf current**

$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar \omega} e^{-\Delta/\kappa T}$$

**DOS peaks oscillate (animation)**

Mattis Bardeen’s formula

$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar \omega} e^{-\Delta/\kappa T}$$

comes from this DOS

**However, it is well known that the DOS is affected by the pair-breaking current.**

Broadening of DOS peaks causes the Q rise

The extended Q-rise is not an exotic but the behavior which follows from the BCS model with the idealized DOS!
Other mechanisms which broaden DOS also affect $R_s$.

→ We Incorporated effects of pair-breaking mechanisms originating from realistic material features into $R_s$ at the weak-field limit.

- Subgap states originating from a finite quasiparticle lifetime.
- Proximity coupled thin Normal layer on the surface
- Small density of magnetic impurities
Other mechanisms which broaden DOS also affect $R_s$.

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These structures model realistic surfaces of superconducting materials which can contain oxide layers, hydrides, absorbed impurities or nonstoichiometric composition.
Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)


DOS of Normal conductor

DOS of BCS superconductor
Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)


The proximity effect changes DOS
**Review (2)**

**Example: Proximity-coupled thin N layer (metallic suboxides)**

Parameters are sensitive to material processing!

- $d$ is an N layer thickness. (e.g., thickness of suboxide on the Nb surface)
- $R_B$ is an interface resistance
  - Sensitive to heat treatment (e.g., between Nb suboxide and Nb)
  - Ref: The lowest contact resistance of YBCO/Ag is $R_B \sim 10^{-13} \Omega \cdot m^2$

We can calculate DOS by using the well-established method: Quasiclassical Green’s function formalism of the BCS theory.
Example: Proximity-coupled thin N layer (metallic suboxides)

The proximity effect changes DOS

As $\beta$ increases, the minigap decreases
Example: Proximity-coupled thin N layer (metallic suboxides)

As $\beta$ increases, the minigap decreases.

The proximity effect changes DOS.

\[ \alpha = 0.05 \]

\[ \beta = \frac{4e^2}{\hbar} R_B N_n \Delta d, \]

\[ \alpha = \frac{N_n}{N_s} \frac{d}{\xi_S} \]

N-side DOS

N-side DOS

As $\beta$ increases, the minigap decreases.
Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)

As $\beta$ increases, the minigap decreases.
**Review (2)**

**Example: Proximity-coupled thin N layer (metallic suboxides)**

\[ \alpha = 0.05 \]

The proximity effect changes DOS ~thickness ~barrier between N&S

\[ \frac{N_n}{N_s} \]

\[ \beta = \frac{4e^2}{h} R_B N_n \Delta d, \]

\[ N(\epsilon)/N_n \]

\[ \epsilon/\Delta \]

\[ 0 \]

\[ 0.5 \]

\[ 1 \]

\[ 1.5 \]

\[ N(\epsilon)/N_s \]

\[ \epsilon/\Delta \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 0.02 \]

\[ 0.04 \]

\[ 0.06 \]

\[ 0.08 \]

\[ 0.1 \]

\[ 0.12 \]

\[ \alpha = 0.05 \]

\[ \beta = 0.1 \]

\[ \beta = 1 \]

\[ \beta = 10 \]

Taking a finite quasi particle lifetime into account ($\varepsilon \to \varepsilon + i\Gamma$), the cusps are smeared out.

The proximity effect changes DOS

These subgap states contribute to residual resistance at $T \to 0$
**Review (2)**

**Example: Proximity-coupled thin N layer (metallic suboxides)**

- $R_s$ depends on the $N$-layer parameters.
- $R_s$ can be optimized by tuning them.
- $R_s$ can be smaller than $R_s$ for the ideal surface without $N$ layer

\[ \beta = \frac{4e^2}{\hbar} R_B N \Delta d, \]

($\sim$ barrier between N&S) can be changed by heat treatments

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Review (2)

Taking Nb for example,

- $\alpha \propto$ thickness of Nb-suboxides or hydrides on the surface
- $\beta \propto$ the interface resistance between Nb suboxide and Nb

These can be easily affected by material processing recipes.

→ Link to the dependence of $R_s$ on various recipes?

$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d$, ($\sim$ barrier between N&S) can be changed by heat treatments

Changing the temperature of heat treatment, $\beta$ can change and then $R_s$ can change.
Example 2: Magnetic impurities can also broaden DOS peaks

\[ \Gamma_p = \frac{\hbar v_F}{2 \ell_p} \]

\( \ell_p \): mean spacing of magnetic impurities

An appropriate density of magnetic impurities significantly reduce \( R_s \)!

\[ \frac{\Gamma_p}{\Delta} \sim 0.01 \] corresponds to the mean spacing of magnetic impurities

\[ \ell_p \sim \frac{\xi_0}{0.01} \sim 4 \mu m \]
Summary of the review part

Pair-breaking current

Ideal SC

Pair-breaking mechanism originating from realistic material features

Magnetic impurities

Thin N layer

Current broadens DOS and affects $R_s$ → field dependence

Magnetic impurities broaden DOS and affect $R_s$

Proximity effect broadens DOS and affects $R_s$. The N layer properties are sensitive to material processing.
Now we address the nonlinear $R_s (H)$

What is the origin of many different Q-E curves?
Part 2

Effects of realistic material features on the field dependent $R_s(H)$

T. Kubo and A. Gurevich, to be published
Strong rf current

Magnetic impurities

Thin N layer


Strong rf current

Strong rf current

Strong rf current

Incorporate a finite quasiparticle lifetime

Magnetic impurities

Proximity coupled N layer at the surface

Proximity effect
Incorporate a finite quasiparticle lifetime. Magnetic impurities. Proximity coupled N layer at the surface.
DOS under a strong rf field

Effect of a finite quasiparticle lifetime
\[ \Gamma = 0.02 \]
\[ \Gamma_p = 0 \]
\[ H_0 = 0.3H_c \]

Effect of magnetic impurities
\[ \Gamma = 0 \]
\[ \Gamma_p = 0.02 \]
\[ H_0 = 0.3H_c \]
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

Strong rf current

$\hbar \omega = 0.004\Delta$

$k_B T = 0.11\Delta$

$\Gamma = 0.06\Delta$

$\Gamma = 0.04\Delta$

$\Gamma = 0.02\Delta$

$H_0/H_c$

$R_s/R_{MB}$
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The Ideal BCS SC exhibits a deep $R_s$ dip
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The nearly ideal BCS SC exhibits the deep $R_s$ dip. The first $R_s$ rise disappears.
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The $R_s$ dip almost disappears, but the low-field $R_s$ is better than ideal BCS SC due to the DOS broadening effect.
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The $R_s$ dip disappears, but the low-field $R_s$ is better than ideal BCS SC due to the DOS broadening effect.
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The $R_s$ dip disappears. $R_s$ becomes larger.
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime ($\Gamma$ parameter)

The broadening parameter ($\Gamma$) has a significant effect on the field dependent $R_s$. 

\[
\hbar \omega = 0.004\Delta \\
k_B T = 0.11\Delta \\
\Gamma = 0.06\Delta \\
\Gamma = 0.04\Delta \\
\Gamma = 0.02\Delta 
\]
**Field dependent Surface resistance** $R_s(H_0)$

**(2) Effect of magnetic impurities ($\Gamma_p$ parameter)**

Magnetic impurities affect $R_s(H_0)$ in the similar manner as a finite quasi particle lifetime $\Gamma$.

- The $R_s$ dip becomes shallower as $\Gamma_p$ increases.
- The low field $R_s$ for ($\Gamma_p \sim 0.01\Delta$) is much smaller than the ideal BCS superconductor ($\Gamma_p = 0$).
Strong rf current

Ideal SC

Magnetic impurities

Thin N layer


Strong rf current

Incorporate a finite quasiparticle lifetime

Magnetic impurities

Proximity effect

proximity coupled N layer at the surface
DOS under a strong rf field

**Proximity effect**

$d$ is an N layer thickness.
(e.g., thickness of suboxide on the Nb surface)

$R_B$ is an interface resistance
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

\[
\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S},
\]

\[
\beta = \frac{4e^2}{h} \frac{R_B N_n \Delta d}{},
\]

Proximity coupled
N layer

\[
\alpha = 0.05, \quad \beta = 1, \quad \Gamma = 0.005
\]
Field dependent Surface resistance $R_s(H_0)$

for different N-layer thickness

$d$ is an N layer thickness.
(e.g., thickness of suboxide on the Nb surface)

$R_B$ is an interface resistance
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

$\beta = \frac{4e^2}{h} R_B N_n \Delta d,$

Proximity effect

Proximity effect between N&S

Graphical representation:

- $\beta=0.1$
- $\Gamma = 0.005$

No normal layer $\rightarrow \alpha=0$

$H_0/H_C$ vs $R_s(H_0)/R_{NB}$

Curves for different $\alpha$:
- $\alpha=0.1$
- $\alpha=0.05$
- $\alpha=0.02$

Graph showing the variation of $R_s(H_0)/R_{NB}$ with $H_0/H_C$ for different $\alpha$.
As the N-layer thickness increases, the dip becomes shallower and finally disappears: Continuously changes from “N-doping-like” to “EP-like” shape.

$R_B$ is an interface resistance
Sensitive to heat treatment (e.g., between Nb suboxide and Nb)

$\alpha = \frac{N_n}{N_s} \frac{d}{\xi S}$
~thickness

$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d$
~barrier between N&S

Proximity effect

Field dependent Surface resistance $R_s(H_0)$ for different N-layer thickness

$\Gamma = 0.005$

No normal layer $\rightarrow \alpha = 0$

$\beta = 0.1$

$\alpha = 0.02$

$\alpha = 0.05$

$\alpha = 0.1$
Field dependent Surface resistance $R_s(H_0)$ for different N-layer conductivity

$d$ is an N layer thickness.
(e.g., thickness of suboxide on the Nb surface)

$R_B$ is an interface resistance
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

\[
\alpha = \frac{N_n}{N_s} \frac{d}{\xi_s}
\]
\[
\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,
\]

Proximity effect

\[\Gamma = 0.005\]

\[\frac{D_n}{D_s} = 20\]

$\alpha=0.05$
$\beta=1$

$D = \frac{\sigma_n}{2N_0e^2}$

\[\frac{H_0}{H_c}\]
The position of minimum shifts from medium fields to lower fields.

\[ R_B \text{ is an interface resistance} \]
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

\[ \alpha = \frac{N_n}{N_s} \frac{d}{\xi_S} \]

\[ \beta = \frac{4e^2}{h} R_B N_n \Delta d, \]

~thickness

~barrier
between N&S

\[ \Gamma = 0.005 \]

Field dependent Surface resistance \( R_s(H_0) \)

\[ \frac{D_n}{D_S} = 20 \]

\[ \alpha = 0.05 \]

\[ \beta = 1 \]

\[ \Gamma = 0.005 \]

diffusivity \( D = \frac{\sigma_n}{2N_0 e^2} \)
Field dependent Surface resistance $R_s(H_0)$ for different temperatures

$d$ is an N layer thickness

$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$

$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d$

Proximity effect

Sensitive to heat treatment (e.g., between Nb suboxide and Nb)

Different types of temperature dependence appear.

$\sigma_n, \sigma_S$ (b)

$\beta = 1$ and $7$

$H_0/H_c$

$\alpha = 0.05$

Blue: 1.4K for Nb
Red: 2K for Nb
Many different shapes of $R_s(H)$ naturally result from the proximity coupled $N$-$S$ model.

e.g.) Nb suboxides or hydrides
Field dependent surface resistance of

- Nb-Nb structure
- Nb₃Sn-Nb structure

T. Kubo and A. Gurevich, to be published
One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$.

Magnetic field distribution

Current density distribution

One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$

This enhances the field limit

(E.g.) NbN-Nb

Current density distribution

One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$

This enhances the field limit and should affects $R_s(H)$ too.

Let’s see what happens to $R_s(H)$

![Current density distribution](image)

One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$.

Example 1: Nb-Nb structure

Dirtier Nb ($\lambda' = 2\lambda$)  
Bulk Nb ($\lambda$)
One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$.

**Example 1: Nb-Nb structure**

$$d = 2\lambda'$$

- **Dirtier Nb** ($\lambda' = 2\lambda$)
- **Bulk Nb** ($\lambda$)
One of the most striking effects is the reduction of the current density at the top layer with $\lambda'(>\lambda)$.

**Example 1: Nb-Nb structure**

\[ d = \lambda' \]

Dirtier Nb
\( (\lambda' = 2\lambda) \)

Bulk Nb
\( (\lambda) \)

![Diagram showing current density and graphs of $R_s(H_0)/R_s(d=0)$ vs $H_0/H_c$ for different values of $\lambda'$.](image)
One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$

**Example 1: Nb-Nb structure**

$$d = 0.5\lambda'$$

Dirtier Nb ($\lambda' = 2\lambda$) ↔ Bulk Nb ($\lambda$)

Current density

![Graph showing current density vs. $H_0/H_c$]
One of the most striking effects is the reduction of the current density at the top layer with $\lambda'(>\lambda)$.

Example 1: Nb-Nb structure

\[ d = 0.25\lambda' \]
One of the most striking effects is the reduction of the current density at the top layer with $\lambda' (> \lambda)$.

**Example 1: Nb-Nb structure**

\[ d = 0 \]
One of the most striking effects is the reduction of the current density at the top layer with $\lambda'(>\lambda)$

**Example 1: Nb-Nb structure**

A dirtier Nb layer with $d \leq \lambda'$ cures the Q slope: $\rightarrow$ “baking-like” effect.
Example 2: Nb$_3$Sn-Nb structure

$d = 0$

Bulk Nb
$(\lambda)$
$\Gamma = 0.01$
**Example 2: Nb$_3$Sn-Nb structure**

\[ d = 0.33\lambda' \]

- **Nb$_3$Sn**  
  \( (\lambda' = 3\lambda) \)  
  \( \Gamma' = 0.05 \)
- **Bulk Nb**  
  \( (\lambda) \)  
  \( \Gamma = 0.01 \)
Example 2: Nb$_3$Sn-Nb structure

\[ d = \lambda' \]

\[ \begin{align*}
\text{Nb}_3\text{Sn} & : (\lambda' = 3\lambda) \\
\Gamma' & = 0.05 \\

\text{Bulk Nb} & : (\lambda) \\
\Gamma & = 0.01
\end{align*} \]
Example 2: $\text{Nb}_3\text{Sn}-\text{Nb}$ structure

$d = 2\lambda'$

$\text{Nb}_3\text{Sn}$
$(\lambda' = 3\lambda)$
$\Gamma' = 0.05$

$\text{Bulk Nb}$
$(\lambda)$
$\Gamma = 0.01$
Example 2: Nb$_3$Sn-Nb structure

The optimum thicknesses for higher fields are given by $d \sim \lambda'$

Here, $Q$ drops are not significant even at $H>H_c^{(Nb)}$
We have developed a theory of **field dependent surface resistance** of a dirty superconductor in a strong RF field, **taking into account realistic materials features** based on the BCS theory.

Many different field dependencies $R_s(H_0)$ naturally result from **realistic material features** such as a finite $\Gamma$, $\Gamma_p$ or a thin N or S layer on the surface.

The surface resistance **can be minimized** by engineering optimum impurity concentration or properties of the surface normal layer.

To compare the theory with experiments and to utilize the theoretical consequences to improve cavity performances, **we need measurements of multiple parameters characterizing a particular material** (e.g., $d$ and $\sigma_n$ of the N layer, $R_B$, and $\Gamma$ parameters) as well as the way these parameters change after different materials treatments.
Wait!
How about nonequilibrium effects on $Rs(H)$???
Part 3

Non-equilibrium effects under strong RF current

T. Kubo and A. Gurevich, to be published
The equation of motion for the Green’s function in Keldysh-Nambu space for a dirty superconductor:

\[ i\hbar \frac{D}{\partial \tau} \otimes [\tilde{G} \otimes (\tilde{\partial} \otimes \tilde{G})] = \left( \tilde{\tau}_z \hbar \frac{\partial}{\partial \tau} + \tilde{\Delta} - \tilde{\Sigma} \right) \otimes \tilde{G} - \tilde{G} \otimes \left( \tilde{\tau}_z \hbar \frac{\partial}{\partial \tau} + \tilde{\Delta} - \tilde{\Sigma} \right) \]

\[ \tilde{G}(\mathbf{R}, t_1, t_2) = \begin{pmatrix} \hat{G}^R \\ 0 \\ \hat{G}^A \end{pmatrix} \]

Consider the first order of slow variation and equilibrium phonons:

We can evaluate non-equilibrium effects under the strong RF current on the distribution function:

\[ f(\epsilon) = \tanh \frac{\epsilon}{2k_B T} \text{ Equilibrium (Fermi-Dirac)} \]

\[ f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t) \text{ non-equilibrium effect} \]

\[ \delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}} \]
- RF frequency ~ 1GHz
- $T = 4K$
- Broadening parameter $\Gamma = 0.05$

Can you see the oscillation? Its effect is very small.

Distribution function is oscillating with RF (non-equilibrium effect)

$$f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t)$$

$$\delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}}$$
- RF frequency \( \sim 1 \text{GHz} \)
- \( T = 2K \)
- Broadening parameter \( \Gamma = 0.1 \)

\[
f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t)
\]

\[
\delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}}
\]
Nonequilibrium effects in $R_s(H)$ becomes significant at lower temperature, higher frequencies, and higher fields.

We started to address this problem.

Look forward to next conferences.
We have developed a theory of field dependent surface resistance of a dirty superconductor in a strong RF field, taking into account realistic materials features based on the BCS theory.

Many different field dependencies $R_s(H_0)$ naturally result from realistic material features such as a finite $\Gamma$, $\Gamma_p$ or a thin N or S layer on the surface.

The surface resistance can be minimized by engineering optimum impurity concentration or properties of the surface normal layer.

To compare the theory with experiments and to utilize the theoretical consequences to improve cavity performances, we need measurements of multiple parameters characterizing a particular material (e.g., $d$ and $\sigma_n$ of the N layer, $R_B$, and $\Gamma$ parameters) as well as the way these parameters change after different materials treatments.

Non-equilibrium effects becomes significant at lower temperatures, higher frequencies and higher fields. We have already started to address this problem. Look forward to SRF2021!